APPENDIX

식 (3)의 정리

$$ETC(N)=\int_0^T [c_1H(y)+c_2]\,dG^N(y)+\int_T^\infty [c_1H(T)+c_2]\,dG^N(y)=c_1\left(\int_0^T H(y)dG^N(y)+\int_T^\infty H(T)dG^N(y)\right)+c_2$$

$$=c_1\left(\int_0^T\int_o^y dH(t)dG^N(y)+\int_T^\infty\int_o^T dH(t)dG^N(y)\right)+c_2=c_1\left(\int_0^T\int_t^T dG^N(y)dH(t)+\int_0^T\int_T^\infty dG^N(y)dH(t)\right)+c_2$$

$$=c_1\int_0^T\int_t^\infty dG^N(x)dH(t)+c_2=c_1\int_0^T\big(1-G^N(t)\big)dH(t)+c_2$$

식 (4)의 정리

$$ETR(N)=\int_0^T ydG^N(y)+\int_T^\infty TdG^N(y)=\int_0^T\int_o^y dtdG^N(y)+\int_T^\infty\int_o^{\wedge T} dtdG^N(y)$$

$$=\int_0^T\int_t^T dG^N(y)dt\ +\int_0^T\int_T^\infty dG^N(y)dt=\int_0^T\left(\int_t^T dG^N(t)+\int_T^\infty dG^N(t)\right)dt$$

$$=\int_0^T\int_t^\infty dG^N(t)dt=\int_0^T\big(1-G^N(t)\big)dt$$

식 (6)의 정리

$$ETCR(N+1)-ETCR(N)=\frac{c_1\int_0^T\big(1-G^{N+1}(t)\big)dH(t)+c_2}{\int_0^T\big(1-G^{N+1}(t)\big)dt}-\frac{c_1\int_0^T\big(1-G^N(t)\big)dH(t)+c_2}{\int_0^T\big(1-G^N(t)\big)dt}$$

$$=\frac{\left(c_1\int_0^T\big(1-G^{N+1}(t)\big)dH(t)+c_2\right)\int_0^T\big(1-G^N(t)\big)dt}{\int_0^T\big(1-G^{N+1}(t)\big)dt\int_0^T\big(1-G^N(t)\big)dt}-\frac{\left(c_1\int_0^T\big(1-G^N(t)\big)dH(t)+c_2\right)\int_0^T\big(1-G^{N+1}(t)\big)dt}{\int_0^T\big(1-G^{N+1}(t)\big)dt\int_0^T\big(1-G^N(t)\big)dt}$$

$$=\frac{c_1\int_0^T\big(1-G^{N+1}(t)\big)dH(t)\int_0^T\big(1-G^N(t)\big)dt-c_1\int_0^T\big(1-G^N(t)\big)dH(t)\int_0^T\big(1-G^N(t)\big)dt}{\int_0^T\big(1-G^{N+1}(t)\big)dt\int_0^T\big(1-G^N(t)\big)dt}$$

$$-\frac{c_1\int_0^T\big(1-G^N(t)\big)dH(t)\left(\int_0^T\big(1-G^{N+1}(t)\big)dt-\int_0^T\big(1-G^N(t)\big)dt\right)-c_2\int_0^T\big(G^N(t)-G^{N+1}(t)\big)dt}{\int_0^T\big(1-G^{N+1}(t)\big)dt\int_0^T\big(1-G^N(t)\big)dt}$$

$$=\left\{\frac{\int_0^T e^{-\theta t}\frac{(\theta t)^N}{N!}dH(t)}{\int_0^T e^{-\theta t}\frac{(\theta t)^N}{N!}dt}\int_0^T\big(1-G^N(t)\big)dt-\int_0^T\big(1-G^N(t)\big)dH(t)-\frac{c_2}{c_1}\right\}c_1\frac{c_1\int_0^T\big(G^N(t)-G^{N+1}(t)\big)dt}{\int_0^T\big(1-G^{N+1}(t)\big)dt\int_0^T\big(1-G^N(t)\big)dt}$$

위의 마지막 등식은 다음에 의해서 성립한다

$$G^N(t)-G^{N+1}(t)=P\{N(t)\geq N\}-P\{N(t)\geq N+1\}=\sum_{n=N}^{\infty}e^{-\theta t}\frac{(\theta t)^n}{n!}-\sum_{n=N+1}^{\infty}e^{-\theta t}\frac{(\theta t)^n}{n!}=e^{-\theta t}\frac{(\theta t)^N}{N!}$$

Lemma 1-1의 증명

$$\frac{\int_0^T e^{-\theta t}\frac{(\theta t)^{N+1}}{(N+1)!}dH(t)}{\int_0^T e^{-\theta t}\frac{(\theta t)^{N+1}}{(N+1)!}dt}-\frac{\int_0^T e^{-\theta t}\frac{(\theta t)^N}{N!}dH(t)}{\int_0^T e^{-\theta t}\frac{(\theta t)^N}{N!}dt}=\frac{\int_0^T e^{-\theta t}(\theta t)^{N+1}dH(t)}{\int_0^T e^{-\theta t}(\theta t)^{N+1}dt}-\frac{\int_0^T e^{-\theta t}(\theta t)^N dH(t)}{\int_0^T e^{-\theta t}(\theta t)^N dt}$$

$$=\frac{\int_0^T e^{-\theta t}(\theta t)^{N+1}dH(t)\int_0^T e^{-\theta t}(\theta t)^N dt-\int_0^T e^{-\theta t}(\theta t)^N dH(t)\int_0^T e^{-\theta t}(\theta t)^{N+1}dt}{\int_0^T e^{-\theta t}(\theta t)^{N+1}dt\int_0^T e^{-\theta t}(\theta t)^N dt}$$

고장률 $h(t)\equiv\frac{f(t)}{\overline{F}(t)}$은 t에 따라서 증가하므로, 모든 $t< T$에 대하여 $h(T)-h(t)>0$이 성립한다. 그러므로

$$\frac{d}{dT}\left[\int_0^T e^{-\theta t}(\theta t)^{N+1}dH(t)\int_0^T e^{-\theta t}(\theta t)^N dt-\int_0^T e^{-\theta t}(\theta t)^N dH(t)\int_0^T e^{-\theta t}(\theta t)^{N+1}dt\right]$$

$$=e^{-\theta T}(\theta T)^{N+1}h(T)\int_0^T e^{-\theta t}(\theta t)^N dt+\int_0^T e^{-\theta t}(\theta t)^{N+1}dH(t)e^{-\theta T}(\theta T)^N-e^{-\theta T}(\theta T)^N h(T)\int_0^T e^{-\theta t}(\theta t)^{N+1}dt$$
$$-\int_0^T e^{-\theta t}(\theta t)^N dH(t)e^{-\theta T}(\theta T)^{N+1}$$

$$=e^{-\theta T}(\theta T)^N\left\{\theta T h(T)\int_0^T e^{-\theta t}(\theta t)^N dt+\int_0^T e^{-\theta t}(\theta t)^{N+1}dH(t)-h(T)\int_0^T e^{-\theta t}(\theta t)^{N+1}dt-\int_0^T e^{-\theta t}(\theta t)^N dH(t)\theta T\right\}$$

$$=e^{-\theta T}(\theta T)^N\left\{h(T)\int_0^T e^{-\theta t}(\theta t)^N[\theta T-\theta t]dt-\int_0^T e^{-\theta t}(\theta t)^N[\theta T-\theta t]dH(t)\right\}$$

$$=e^{-\theta T}(\theta T)^N\int_0^T e^{-\theta t}(\theta t)^N[\theta T-\theta t][h(T)-h(t)]dt>0$$

그러므로 $\frac{\int_0^T e^{-\theta t}\frac{(\theta t)^{N+1}}{(N+1)!}dH(t)}{\int_0^T e^{-\theta t}\frac{(\theta t)^{N+1}}{(N+1)!}dt}-\frac{\int_0^T e^{-\theta t}\frac{(\theta t)^N}{N!}dH(t)}{\int_0^T e^{-\theta t}\frac{(\theta t)^N}{N!}dt}$은 N이 증가함에 따라서 증가한다.

Lemma 1-2의 증명

$$\left[\frac{\int_0^T e^{-\theta t}\frac{(\theta t)^{N+1}}{(N+1)!}dH(t)}{\int_0^T e^{-\theta t}\frac{(\theta t)^{N+1}}{(N+1)!}dt}\int_0^T\left(1-G^{N+1}(t)\right)dt-\int_0^T\left(1-G^{N+1}(t)\right)dH(t)-\frac{c_2}{c_1}\right]$$

$$-\left[\frac{\int_0^T e^{-\theta t}\frac{(\theta t)^{N}}{N!}dH(t)}{\int_0^T e^{-\theta t}\frac{(\theta t)^{N}}{N!}dt}\int_0^T\left(1-G^{N}(t)\right)dt-\int_0^T\left(1-G^{N}(t)\right)dH(t)-\frac{c_2}{c_1}\right]$$

$$=\frac{\int_0^T e^{-\theta t}(\theta t)^{N+1}dH(t)}{\int_0^T e^{-\theta t}(\theta t)^{N+1}dt}\int_0^T\left(1-G^{N+1}(t)\right)dt-\int_0^T\left(1-G^{N+1}(t)\right)dH(t)$$

$$-\frac{\int_0^T e^{-\theta t}(\theta t)^{N}dH(t)}{\int_0^T e^{-\theta t}(\theta t)^{N}dt}\int_0^T\left(1-G^{N}(t)\right)dt+\int_0^T\left(1-G^{N}(t)\right)dH(t)$$

$$=\frac{\int_0^T e^{-\theta t}(\theta t)^{N+1}dH(t)}{\int_0^T e^{-\theta t}(\theta t)^{N+1}dt}\left(\int_0^T\left(1-G^{N+1}(t)\right)dt-\int_0^T\left(1-G^{N}(t)\right)dt\right)+\frac{\int_0^T e^{-\theta t}(\theta t)^{N+1}dH(t)}{\int_0^T e^{-\theta t}(\theta t)^{N+1}dt}\int_0^T\left(1-G^{N}(t)\right)dt$$

$$-\frac{\int_0^T e^{-\theta t}(\theta t)^{N}dH(t)}{\int_0^T e^{-\theta t}(\theta t)^{N}dt}\int_0^T\left(1-G^{N}(t)\right)dt-\int_0^T\left(G^{N}(t)-G^{N+1}(t)\right)dH(t)$$

$$=\frac{\int_0^T e^{-\theta t}(\theta t)^{N+1}dH(t)}{\int_0^T e^{-\theta t}(\theta t)^{N+1}dt}\int_0^T\left(G^{N}(t)-G^{N+1}(t)\right)dt+\left(\frac{\int_0^T e^{-\theta t}(\theta t)^{N+1}dH(t)}{\int_0^T e^{-\theta t}(\theta t)^{N+1}dt}-\frac{\int_0^T e^{-\theta t}(\theta t)^{N}dH(t)}{\int_0^T e^{-\theta t}(\theta t)^{N}dt}\right)\int_0^T\left(1-G^{N}(t)\right)dt$$

$$-\int_0^T\left(G^{N}(t)-G^{N+1}(t)\right)dH(t)$$

$$=\int_0^T\left(G^{N}(t)-G^{N+1}(t)\right)dt\left(\frac{\int_0^T e^{-\theta t}(\theta t)^{N+1}dH(t)}{\int_0^T e^{-\theta t}(\theta t)^{N+1}dt}-\frac{\int_0^T\left(G^{N}(t)-G^{N+1}(t)\right)dH(t)}{\int_0^T\left(G^{N}(t)-G^{N+1}(t)\right)dt}\right)$$

$$+\left(\frac{\int_0^T e^{-\theta t}(\theta t)^{N+1}dH(t)}{\int_0^T e^{-\theta t}(\theta t)^{N+1}dt}-\frac{\int_0^T e^{-\theta t}(\theta t)^{N}dH(t)}{\int_0^T e^{-\theta t}(\theta t)^{N}dt}\right)\int_0^T\left(1-G^{N}(t)\right)dt$$

$$=\left(\frac{\int_0^T e^{-\theta t}(\theta t)^{N+1}dH(t)}{\int_0^T e^{-\theta t}(\theta t)^{N+1}dt}-\frac{\int_0^T e^{-\theta t}(\theta t)^{N}dH(t)}{\int_0^T e^{-\theta t}(\theta t)^{N}dt}\right)\int_0^T\left(1-G^N(t)\right)dt > 0$$

그러므로 $\frac{\int_0^T e^{-\theta t}\frac{(\theta t)^N}{N!}dH(t)}{\int_0^T e^{-\theta t}\frac{(\theta t)^N}{N!}dt}\int_0^T\left(1-G^N(t)\right)dt-\int_0^T\left(1-G^N(t)\right)dH(t)-\frac{c_2}{c_1}$은 N이 증가함에 따라서 증가한다.